

Quiz 6

September 16, 2016

Show all work and circle your final answer.

1. Find all asymptotes (vertical and horizontal) of $y = \frac{\sqrt{2x^2 - x + 1}}{x - 1}$.

$$\begin{aligned} \text{HAs: } \lim_{x \rightarrow \infty} \frac{\sqrt{2x^2 - x + 1}}{x - 1} \cdot \frac{1/\sqrt{x^2}}{1/x} &= \lim_{x \rightarrow \infty} \frac{\sqrt{2 - 1/x + 1/x^2}}{1 - 1/x} = \sqrt{2} \\ \lim_{x \rightarrow -\infty} \frac{\sqrt{2x^2 - x + 1}}{x - 1} \cdot \frac{1/\sqrt{x^2}}{1/x} &= \lim_{x \rightarrow -\infty} \frac{\sqrt{2 - 1/x + 1/x^2}}{1 - 1/x} = -\sqrt{2} \end{aligned}$$

VA's: $x - 1 = 0$ when $x = 1$, and $2(1)^2 - 1 + 1 \neq 0$

$$\begin{aligned} y &= \sqrt{2} \\ y &= -\sqrt{2} \\ x &= 1 \end{aligned}$$

2. Use the limit definition of the derivative to find $f'(a)$ if $f(x) =$

$$\frac{3x}{x+1}$$

$$\begin{aligned} f'(a) &= \lim_{x \rightarrow a} \frac{\frac{3x}{x+1} - \frac{3a}{a+1}}{x-a} \\ &= \lim_{x \rightarrow a} \frac{3x(a+1) - 3a(x+1)}{(x-a)(x+1)(a+1)} \quad \text{OR} \\ &= \lim_{x \rightarrow a} \frac{3(x-a)}{(x-a)(x+1)(a+1)} \\ &= \boxed{\frac{3}{(a+1)^2}} \end{aligned}$$

$$\begin{aligned} f'(a) &= \lim_{h \rightarrow 0} \frac{\frac{3(a+h)}{a+h+1} - \frac{3a}{a+1}}{h} \\ &= \lim_{h \rightarrow 0} \frac{3(a+h)(a+1) - 3a(a+h+1)}{h(a+h+1)(a+1)} \\ &= \lim_{h \rightarrow 0} \frac{3a^2 + 3a + 3ah + 3h - 3a^2 - 3ah - 3a}{h(a+h+1)(a+1)} \\ &= \lim_{h \rightarrow 0} \frac{3h}{h(a+h+1)(a+1)} = \boxed{\frac{3}{(a+1)^2}} \end{aligned}$$

3. Sketch a graph of the derivative of the function below.

